Suite exos 1,

**Exercice 4.14.** Trouver le chiffre des unités de $\boxed{109}^{2\,007}$ et $23^{24^{25}}$.

$109^k \equiv 9^k \, [10]$

| $K$ | $109^K$ mod 10. |
|---|---|
| 1 | $109 \equiv 9$ |
| 2 | $9 \times 9 \equiv 1$ |
| 3 | $\boxed{\equiv} 9.$ |
| 4 | $\equiv 1$ |

$23 \equiv 3 \mod 10.$

| $K$ | mod 10 |
|---|---|
| 1. | $23 \equiv 3$ |
| 2 | $23 \times 23 \equiv 3 \times 3 \equiv 9.$ |
| 3 | $23^2 \times 23 \equiv 9 \times 3 \equiv 7$ |
| 4 | $23^3 \times 23 \equiv 7 \times 3 \equiv 1$ |
| 5 | $23^4 \times 23 \equiv 1 \times 3 \equiv 3.$ |

→ cycle de 4.

$24 \equiv 0 \, [4].$

car $6 \times 4 = 24.$

$23^{24} \equiv 1 \mod 10.$

$(23^{24})^{25} \equiv 1^{25} \mod 10.$

(2) $123x \equiv 7 \mod 5$

$123 \equiv 7 \mod 5$ a une solt° $\Longleftrightarrow$ PGCD$(123 ; 5) \mid 7$

$123 = 5 \times 24 + 3$ $\quad 1 \mid 7$. ok.

$5 = 3 \times 1 + 2$

$3 = 2 \times 1 + 1$

$2 = 1 \times 2 + 0$,

$1 = 3 - 2 \times 1$

$= 3 - (5 - 3 \times 1) \times 1 = 2 \times 3 - 5 \times 1.$

$= \quad 2 \times (123 - 5 \times 12) - 5 \times 1$

$= 2 \times 123 - 5 \times 48 - 5 \times 1.$

$2 \times 123 - 49 \times 5.$

$x_0 = 2, \ y_0 = -49.$

$c' = \frac{7}{1}$ ; $a' = a$, $b' = b$.

$x - 7 \times 2 = 5k$ ; $y + 49 \times 2 = -123k$.

$x = c' x_0 + b' k$

$\Rightarrow x = 7 \times 2 + 5k \Rightarrow x = 5k + 14$