

TD32: Maths.

$$\tan(a+b) = \frac{\tan(a) + \tan(b)}{1 + \tan(a)\tan(b)}$$

$$f(x) = \arctan(2x) + \arctan(x) = \frac{\pi}{4}$$

$$\tan(\arctan(2x) + \arctan(x)) = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{\tan(\arctan(2x)) + \tan(\arctan(x))}{1 + \tan(\arctan(2x))\tan(\arctan(x))} = 1$$

$$\frac{2x + x}{1 + 2x^2} = 1 \Rightarrow 3x = 1 + 2x^2$$

$$\arctan(2x)$$

$$\Rightarrow 2x^2 - 3x + 1 = 0$$

$$\Rightarrow 2x \in \mathbb{R}$$

$$\Delta = 9 - 4 = 5, \quad x_1 = \frac{3+1}{4}, \quad x_2 = \frac{3-1}{4} = \frac{1}{2}$$

$$x \in \mathbb{R}$$

$$3^{2x+1} + 3^{2x} = 4^{x+1} + 4^{x+\frac{1}{2}}$$

$$3 \times 3^{2x} + 3^{2x} = 4^x \cdot 4 + 4^x \sqrt{4}$$

$$= 3^{2x}(3+1) = 4^x(4+2) \text{ or } 3^{2x}(3+1) = 4^x(4+2)$$

$$4 \times 9^x = 6 \times 4^x$$

$$\Rightarrow \frac{9^x}{4^x} = \frac{6}{4} \Rightarrow \left(\frac{9}{4}\right)^x = \frac{6}{4} \Rightarrow x(\ln(9) - \ln(4)) = \ln(6) - \ln(4)$$

$$\Rightarrow x = \frac{\ln(3 \cdot 2) - 2\ln(2)}{2(\ln(3) - \ln(2))} = \frac{\ln(3) + \ln(2) - 2\ln(2)}{2(\ln(3) - \ln(2))} = \frac{1}{2}$$

$$\arccos(x) = \arctan\left(\frac{\sqrt{(1+x)(1-x)}}{x}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{1-x^2}} = \frac{\left(\frac{\sqrt{(1+x)(1-x)}}{x}\right)'}{1 + \frac{(x+1)(x-1)}{x^2}} = \frac{\frac{2x}{x\sqrt{(1+x)(1-x)}} + \frac{x\sqrt{(1+x)(1-x)}}{x^2}}{1 + \frac{(x+1)(x-1)}{x^2}} = \frac{\frac{2}{\sqrt{(1+x)(1-x)}} + \frac{x}{x\sqrt{(1+x)(1-x)}}}{1 + (x+1)(x-1)}$$

$$= - \frac{z + z(1+z)(1-z)}{\sqrt{(1+z)(1-z)}} = - \frac{(z + z(1+z)(1-z)) (1 \mp 1-z^2)}{\sqrt{1-z^2}}$$
$$1 + (z+1)(z-1)$$

