TD 34 : Maths.

$(u_n)_{n\in\mathbb{N}}$

$u_0 = 0$

$u_{n+1} = \sqrt{u_n + 2}$.

calculer u_1 et u_2 $\quad u_1 = \sqrt{2}$; $u_2 = \sqrt{\sqrt{2}+2}$.

Mq $f(x) = \sqrt{x+2}$ ↗ sur $\mathbb{R}_+$

$$f'(x) = \frac{1}{2\sqrt{x+2}} > 0 \quad \forall x \in \mathbb{R}_+$$

$\forall x \in [0;2]$, $\sqrt{x+2} \leq 2$.

$0 \leq x \leq 2$. $\Rightarrow f(0) \leq f(x) \leq f(2)$ car f ↗ sur $[0;2]$.

$\Rightarrow \sqrt{2} \leq \sqrt{x+2} \leq 2$. $\Rightarrow \sqrt{x+2} \leq 2$. ok.

Mq. $\forall n \in \mathbb{N}$, $u_{n+1} - u_n > 0$

Soit P_n : $u_{n+1} - u_n \geq 0$. Montrons P_n $\forall n \in \mathbb{N}$ par récurrence.

P_0 : $u_1 - u_0 = \sqrt{2} - 0 > 0$.

On suppose P_n vrai pour un n quelconque. Montrons P_{n+1}.

$u_{n+1} - u_n > 0$. $\Rightarrow u_{n+1} \geq u_n$

$= f(u_{n+1}) \geq f(u_n)$ (f st ↗.

$u_{n+2} \geq u_{n+1}$.

Hérédité vrai donc $\forall n \in \mathbb{N}$, (u_n) ↗.

(u_n) est croissante et majorée donc $(u_n)_{n\in\mathbb{N}}$ tend vers un Réel ℓ.

$\ell = \sqrt{\ell + 2} \Rightarrow \ell^2 = \ell + 2 \Rightarrow \ell^2 - \ell - 2 = 0 \quad \Delta = 1 + 8 = 9$.

$\ell_1 = \frac{1+3}{2} = 2$. valide car $u_n \geq 0 \ \forall n \in \mathbb{N}$.

Donc $(u_n)_{n\in\mathbb{N}}$ converge vers 2.

$$\arctan(x) + \arctan(\sqrt{3}x) = \frac{7\pi}{12}$$

$$\tan(\arctan(x) + \arctan(\sqrt{3}x)) = \tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \frac{\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}.$$

$$\tan(\arctan(x) + \arctan(\sqrt{3}x)) = \frac{1+\sqrt{3}}{1-\sqrt{3}}.$$

$$\frac{\cancel{\tan(\arctan}(x)) + \cancel{\tan(\arctan}(\sqrt{3}x))}{1-\cancel{\tan(\arctan}(x))\cancel{\tan}(\arctan(\sqrt{3}x))} = \frac{\cancel{x}(1+\sqrt{3})}{x^4\sqrt{3}} = \frac{x(1+\sqrt{3})}{1-x^2\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}}.$$

Par comparaison, $x = 1$.

$$\arcsin(2x) - \arcsin(x\sqrt{3}) = \arcsin(x).$$

$$\sin(\arcsin(2x) - \arcsin(x\sqrt{3})) = x.$$

$$= \sin(\arcsin(2x))\cos(\arcsin(x\sqrt{3})) - \sin(\arcsin(x\sqrt{3}))\cos(\arcsin(2x))$$

A FINIR.

$$\arccos(x) = \arctan\left(\frac{\sqrt{(1+x)(1-x)}}{x}\right)$$

$$\arccos'(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\arctan'\left(\frac{\sqrt{1+x}\sqrt{1-x}}{x}\right) = \frac{1}{1+\left(\frac{\sqrt{(1-x)(1+x)}}{x}\right)^2} \times \left(\frac{\sqrt{(1+x)(1-x)}}{x}\right)'$$

$$= \frac{\cancel{x^2}}{2x^2-1}\left(\frac{\frac{2x^2}{2\sqrt{(1+x)(1+x)}} - \sqrt{(1+x)(1-x)}}{\cancel{x^2}}\right) = \left(\frac{1}{\sqrt{(1-x)(1+x)}} - \frac{\sqrt{(1+x)(1-x)}^2}{\sqrt{(1+x)(1+x)}}\right) = \frac{\cancel{x^2}-\cancel{x^2}-1}{\sqrt{x^2-1}} \text{ ok.}$$