TD 11 : Maths

Exercice 5.6. Calculer les primitives suivantes :

$$\int \frac{1}{x^2+2x-3}\,dx\,,\quad \int \frac{x}{x^2-2x-3}\,dx\,,\quad \int \frac{x+1}{x(x+2)}\,dx\,,\quad \int \frac{x+1}{x^2-x-6}\,dx\,,$$

$$\int \frac{x}{x^2-2x+2}dx\,,\quad \int \frac{x+1}{x^2+4}\,dx\,,\quad \int \frac{x+1}{x^2+x+1}\,dx\,,\quad \int \frac{1}{(x+1)(x^2+1)}\,dx.$$

2 $\displaystyle\int \frac{x}{x^2-2x-3}\,dx$ $\quad\|\quad x^2-2x-3=(x-3)(x+1)$

$\dfrac{x}{(x-3)(x+1)} = \dfrac{A}{x-3}+\dfrac{B}{x+1}$ $\quad \dfrac{x}{x-3} = \dfrac{A\cancel{(x+1)}}{\cancel{x-3}}+B$ $\quad x=-1.\ B=\dfrac{1}{4}.$

$\dfrac{x}{x+1}=A$ en $x=3$ $\quad A=\dfrac{3}{4}.$

$=\dfrac{3}{4}\displaystyle\int\frac{1}{x-3}+\frac{1}{4}\int\frac{1}{x+1} = \frac{3}{4}\ln(x-3)+\frac{1}{4}\ln(x+1) =$

3 $\displaystyle\int \frac{x+1}{x(x+2)}\,dx = \frac{1}{2}\int\frac{2x+2}{x^2+2x}\,dx = \frac{1}{2}\ln(2x+2)$

4 $\displaystyle\int\frac{x+1}{x^2-x-6}\,dx$ $\quad\|\quad x^2-x-6=(x-3)(x+2)$

$\dfrac{x+1}{(x-3)(x+2)}=\dfrac{A}{x-3}+\dfrac{B}{x+2}$ $\quad B=\dfrac{1}{5}\ ;\ A=\dfrac{4}{5}$

$=\dfrac{4}{5}\displaystyle\int\frac{1}{x-3}\,dx+\frac{1}{5}\int\frac{1}{x+2}\,dx = \frac{4}{5}\ln(x-3)+\frac{1}{5}\ln(x+2)$

8 $\displaystyle\int\frac{x}{x^2-2x+2}\,dx$ $\quad \Delta<0 \quad (x^2-2x+2)'=2x-2 = 2(x-1)$

$=\dfrac{1}{2}\displaystyle\int\frac{2(x-1)}{x^2-2x+2}\,dx+\int\frac{1}{x^2-2x+2}\,dx$

$$= \frac{1}{2}\ln(x^2-2x+2) + \int \frac{dx}{x^2-2x+2} = \frac{1}{2}\ln(x^2-2x+2) + \int \frac{dx}{1+(x-1)^2} = \frac{1}{2}\ln(x^2-2x+2) + \arctan(x-1).$$

↑ φ canonique.

⑥ $$\int \frac{x+1}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4) + \int \frac{dx}{x^2+4}$$

$$= \frac{1}{2}\ln(x^2+4) + \frac{1}{4}\int \frac{dx}{1+\left(\frac{1}{2}x\right)^2} = \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\arctan\left(\frac{1}{2}x\right)$$

⑦ $$\int \frac{x+1}{x^2+x+1}\,dx = \frac{1}{2}\int \frac{2x+1}{x^2+x+1} + \int \frac{1}{x^2+x+1}\,dx$$

$$1\left(x^2+x+\frac{1}{4}\right) - \frac{1}{4} + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}.$$

$$= \frac{1}{2}\ln(x^2+x+1) + \int \frac{1}{(x+1)^2+\frac{3}{4}} = \frac{1}{2}\ln(x^2+x+1) + \frac{4}{3}\int \frac{dx}{1+\left(\frac{x+1}{\sqrt{3}/2}\right)^2}$$

on pose $u = \frac{2(x+1)}{\sqrt{3}}$.

$du = \frac{2}{\sqrt{3}}dx$

$$= \frac{1}{\sqrt{3}}\int \frac{du}{1+u^2} = \frac{1}{2}\ln(x^2+x+1) + \frac{1}{\sqrt{3}}\arctan\left(\frac{2(x+1)}{\sqrt{3}}\right).$$

⑧ $$\int \frac{dx}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2+1}.$$

$$\frac{A(x^2+1) + (x+1)(Bx+C)}{(x+1)(x^2+1)} = \frac{x^2(A+B) + x(B+C) + A+C}{(x+1)(x^2+1)}$$

$$\begin{cases} A+B=0 \\ B+C=0 \\ A+C=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ B=-C \\ A+C=1 \end{cases} \Rightarrow \begin{cases} A=C=-B \\ 2A=1 \Rightarrow A=\frac{1}{2} \end{cases}$$

$A=\frac{1}{2}$; $B=-\frac{1}{2}$, $C=\frac{1}{2}$

$-\frac{1}{2} \; x - 4 = 2.$

$$= \frac{1}{2}\int \frac{dx}{x+1} + \int \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}\,dx = \frac{1}{2}\ln(x+1) - \frac{1}{2}\int \frac{2x+1}{x^2+1}$$

$$= \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x^2+1) + \frac{1}{2}\arctan(x)$$

$$= \frac{1}{2}\left(\ln(x+1) - \ln(x^2+1) + \arctan(x)\right).$$

Exercice 5.7. (1) Soit $f(x) = \frac{5x^2+21x+22}{(x-1)(x+3)^2}$, $x \in]1,+\infty[$. Calculer les primitives de f sur l'intervalle $]1,+\infty[$.

(2) En déduire la primitive de f sur $]1,+\infty[$ qui s'annule en 2.

$$\frac{5x^2+21x+22}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{5x^2+21x+22}{(x+3)^2} = A \quad x=1. \quad A = \frac{5+21+22}{16} = \frac{48}{16} = \frac{8\times 6}{8\times 2} = 3.$$

$$\frac{5x^2+21x+22}{(x-1)} = C \quad x=-3. \Rightarrow \frac{9\times 5+21(-3)+22}{-4} = \frac{67-63}{-4} = -1 = C.$$

$$x=0 \Rightarrow \frac{22}{-9} = -3 + \frac{1}{3}B - \frac{1}{9} = \frac{22}{-3} = -9 + B - \frac{1}{3} \Rightarrow -7+9 = B \Rightarrow B=2.$$

$$F(x) = \frac{1}{3}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x+3} + \int \frac{-1}{(x+3)^2}\,dx = \frac{1}{3}\ln(x-1) + \frac{1}{2}\ln(x+3) + \frac{1}{x+3} + C.$$

$$F(2)=0 \Rightarrow \cancel{\frac{1}{3}\ln(1)} + 2\ln(5) + \frac{1}{5} + C = 0 \Rightarrow c = -\left(2\ln(5) + \frac{1}{5}\right)$$

Exercice 5.8. Calculer les intégrales suivantes :

$$I = \int_1^2 \frac{3x^2+x+2}{x(4x^2+2)}\,dx,$$

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$$J = \int_0^{\ln(2)} \frac{3e^u+2e^{-u}+1}{4e^u+2e^{-u}}\,du.$$

$$\frac{2}{2} = A \Rightarrow A = 1.$$

(1) $$\int \frac{3x^2+x+2}{x(4x^2+2)}\,dx \qquad \frac{3x^2+x+2}{x(4x^2+2)} = \frac{A}{x} + \frac{Bx+C}{4x^2+2}$$

$$4(4x^2+2x) + x(Bx+C) = x^2(16+B) + x(8+C)$$

$$B=-1; \quad C=1.$$

$$= \int \frac{dx}{x} + \int \frac{x-1}{4x^2+2} =$$

$u = \sqrt{2}x$

$du = \sqrt{2}\,dx$

$$= \ln(x) + \frac{1}{8}\ln(4x^2+2) - \frac{1}{2}\int \frac{1}{2x^2+1}\,dx$$

$$= \ln(x) + \frac{1}{8}\ln(4x^2+2) - \frac{1}{2\sqrt{2}}\int \frac{1}{1+u^2}\,du = \left[\ln(x) + \frac{1}{8}\ln(4x^2+2) - \frac{1}{2\sqrt{2}}\arctan(\sqrt{2}\,x)\right]_1^2$$

$$= \ln(2) + \frac{1}{8}\ln(18) - \frac{1}{2\sqrt{2}}\arctan(2\sqrt{2}) - \cancel{\ln(1)} - \frac{1}{8}\ln(6) \cdot \frac{1}{2\sqrt{2}}\arctan(\sqrt{2})$$

$$= \ln(2) + \frac{1}{8}\ln(108) - \frac{1}{2\sqrt{2}}\left(\arctan(2\sqrt{2}) - \frac{1}{2\sqrt{2}}\arctan(\sqrt{2})\right)$$

(1) $f(x) = \sqrt{\frac{x+1}{x-1}}$, sur l'intervalle $]1, +\infty[$.

$$\int f(x) = \sqrt{\frac{x+1}{x-1}}\,dx \qquad u = \frac{x+1}{x-1} \qquad du = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\,dx = (x+1)\,dx$$

$$\int$$