TD32 : Maths.

Exercice 5.9. En appliquant les règles de Bioche, calculer les intégrales suivantes :

$$I = \int_0^{\pi/4} \frac{\tan(x)}{\sqrt{2}\cos(x) + 2\sin^2(x)}\,dx\,, \quad J = \int_0^{\pi/2} \frac{dx}{2+\sin(x)}\,, \quad L = \int_{\pi/3}^{\pi/2} \frac{dx}{\sin(x)+\sin(2x)}.$$

$$I = \int_0^{\frac{\pi}{4}} \frac{\sin(x)}{\cos(x)\left(\sqrt{2}\cos(x)+2-2\cos^2(x)\right)}\,dx = \int_1^{\sqrt{2}/2} \frac{-du}{u(\sqrt{2}u+2-2u^2)} = \int_1^{\sqrt{2}/2} \frac{-du}{2u(u-\sqrt{2})(u+\frac{\sqrt{2}}{2})}$$

$u = \cos(x)$, $du = -\sin(x)\,dx$

$$\frac{-1}{2u(u-\sqrt{2})(u+\frac{\sqrt{2}}{2})} = \frac{A}{2u} + \frac{B}{u-\sqrt{2}} + \frac{C}{u+\frac{\sqrt{2}}{2}}.$$

$$u=0 \Rightarrow A = \frac{-1}{\cancel{2}(-\sqrt{2})(\frac{\sqrt{2}}{\cancel{2}})} = \frac{1}{2}.$$

$$u=\sqrt{2} \Rightarrow B = \frac{-1}{\cancel{2}\sqrt{2}\,\frac{3\sqrt{2}}{\cancel{2}}} = -\frac{1}{6}$$

$$u = -\frac{\sqrt{2}}{2} \Rightarrow C = \frac{-1}{-\cancel{\sqrt{2}}\left(-\frac{3\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\cancel{\sqrt{2}}}\right)} = -\frac{1}{3}.$$

$$J = \int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin(x)}$$

$$J = \int_0^{\pi/2} \frac{dx}{2+\sin(\frac{\pi}{2}-x)}$$

$$2J = \int_0^{\frac{\pi}{2}} \left(\frac{1}{2+\cos(x)} + \frac{1}{2+\sin(x)}\right) dx$$

$$= \int_0^{\pi/2} \frac{4+\sin(x)+\cos(x)}{(2+\cos(x))(2+\sin(x))}$$

$$= \int_0^{\pi/2} \frac{\sin(x)+\cos(x)+4}{4+\cos(x)\sin(x)+2\sin(x)+2\cos(x)}\,dx$$

$$\sin(x)+\cos(x) = \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos(x)+\frac{\sqrt{2}}{2}\sin(x)\right)$$
$$= \sqrt{2}\left(\cos(\tfrac{\pi}{4})\cos(x)+\sin(\tfrac{\pi}{4})\sin(x)\right) = \sqrt{2}\cos(\tfrac{\pi}{4}-x)$$

$$= \int_0^{\pi/2} \frac{\sin(x)+\cos(x)+4}{2(\sin(x)+\cos(x))+\cos(x)\sin(x)+4}\,dx.$$

???

Exercice 5.10. Calculer les primitives des fonctions suivantes :

(1) $f(x) = \sqrt{\frac{x+1}{x-1}}$, sur l'intervalle $]1, +\infty[$.

(2) $g(x) = \frac{1}{x\sqrt{x^2+x-6}}$, sur l'intervalle $]2, +\infty[$.

Indication : on pourra effectuer le changement de variable $x + \frac{1}{2} = \frac{5}{2}\mathrm{ch}(u)$.

(3) $h(x) = \frac{1}{\sqrt{x}+\sqrt[3]{x}}$, sur l'intervalle $]0, 1[$.

① $f(x) = \sqrt{\frac{x+1}{x-1}}$ $\qquad \int f(x)dx = \int \sqrt{\frac{x+1}{x-1}}\, dx.$

$\frac{x-1+2}{x-1} : \sqrt{1 + \frac{2}{x-1}}$

$u = x - 1 \quad du = dx \Rightarrow \int \sqrt{1 + \frac{2}{u}}\, du$