TD10 : Proba :

Exo supp.

1. (a) facile, $C = \frac{1}{15}$

(c) $p(t) = \frac{1}{15}\int_0^{30} e^{-\frac{t}{15}} dt = \frac{1}{15}\left[-15e^{-\frac{t}{15}}\right]_0^{30} = -e^{-2} + 1.$

La fonct° de répartit° d'une V.A, noté $F_X(t) = \int_{-\infty}^{t} f_X(t)dt.$

Pour la loi exponentielle, $F_X(t) = \begin{cases} 0 \text{ si } t<0 \\ 1-e^{-\lambda t} \text{ si } t \geq 0 \end{cases}$

A = "Attendre 30 minutes" $\quad \mathbb{P}(A) = \mathbb{P}(X \geq 30)$

B = "Attendre 30 minutes de plus" $\quad \mathbb{P}(B) = \mathbb{P}(X \geq 30)$

$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)}$

$\mathbb{P}(A \cap B) = 1 - e^{-4}.$ $\quad \mathbb{P}(A|B) = \frac{e^{-4}}{e^{-2}} = e^{-2} = \mathbb{P}(A) \Rightarrow$ la perte de mémoire de la loi exponentielle.

$\mathbb{E}(t) = \int_{\mathbb{R}} t p(t) dt. = \frac{1}{\lambda}$

$V(T) = \frac{1}{\lambda^2}$

Thm : soit X V.A ≥ 0. Alors :

$$\mathbb{E}(X) = \int_0^{+\infty} \mathbb{P}(X \geq t) dt.$$

On a $X = \min(T, 60)$. Donc

$$E(X) = \int_0^{+\infty} \mathbb{P}(\min(T,60) \geq t)) dt$$

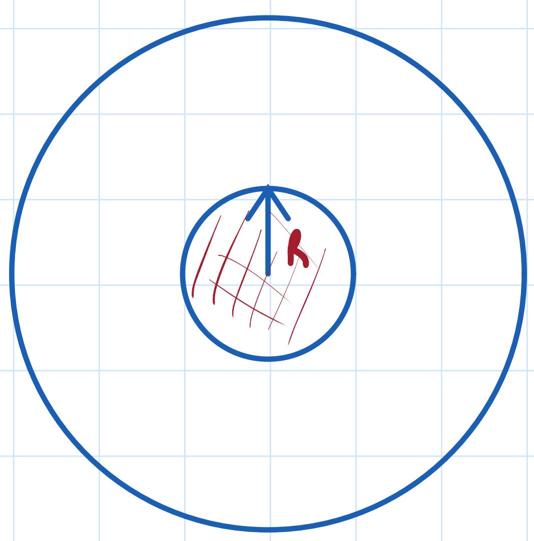
$$= \int_0^{60} \mathbb{P}(T \geq t) dt = \int_0^{60} e^{-\frac{t}{15}} dt = -15\left[e^{-\frac{t}{15}}\right]_0^{60} = -15e^{-4} + 15 = 15(1 - e^{-4}).$$

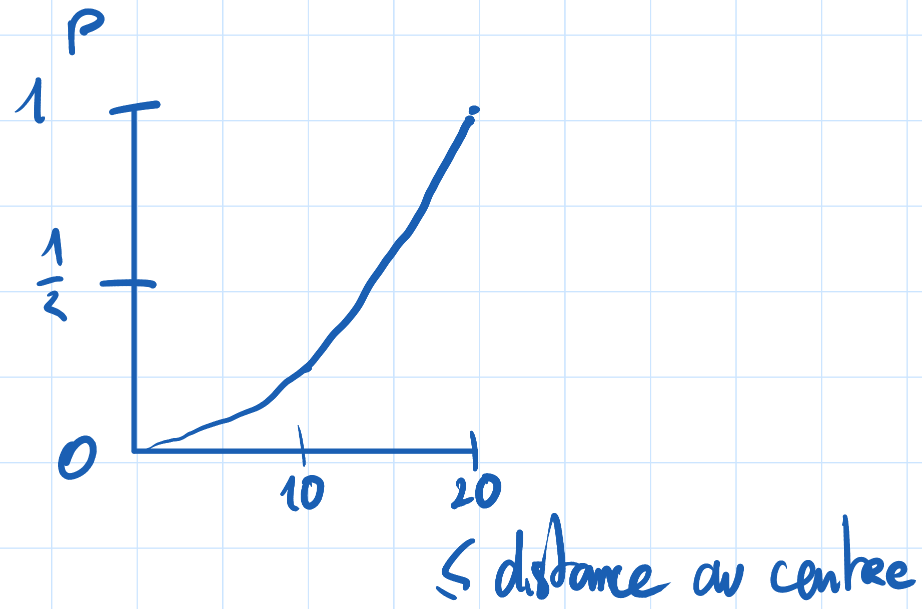
2. a $\int_0^{20} Cx\,dx = \frac{C}{2}\left[x^2\right]_0^{20} = 200\,C = 1 \Rightarrow C = \frac{1}{200}$

(b) $\mathbb{P}(X=0) = 0$

(c) $\mathbb{P}(X \leq 1) = \frac{1}{200}\int_0^1 x\,dx = \frac{1}{200} \times \frac{1}{2}\left[x^2\right]_0^1 = \frac{1}{400}$

(d) $\mathbb{P}(X \leq R) = \frac{1}{200}\int_0^R x\,dx = \frac{1}{400}\left[x^2\right]_0^R = \frac{R^2}{400}$. ok





Surface de la cible : $\pi 10^2 = 100\pi$.

Surface $\mathcal{D}$: $\pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$ $\quad \frac{\mathcal{D}}{S_c} = \frac{\frac{\cancel{\pi} R^2}{4}}{100\cancel{\pi}} = \frac{R^2}{400}$.

→ La loi de proba est uniforme

$\mathbb{E}(X) = \frac{1}{200}\int_0^{20} x^2\,dx = \frac{1}{600}\left[x^3\right]_0^{20} = \frac{1}{600} = \frac{8000}{600} = \frac{40}{3}$

$\mathbb{E}(X^2) = \frac{1}{200}\int_0^{20} x^3\,dx = \frac{1}{800}\left[x^4\right]_0^{20} = \frac{160\,000}{800} = 200$

$\mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 200 - \left(\frac{40}{3}\right)^2 = \frac{1800}{9} - \frac{1600}{9} = \frac{200}{9}$

$\mathbb{P}(X \leq R) = \mathbb{P}(X \geq R)$

$\frac{R^2}{400} = 1 - \frac{R^2}{400} \Rightarrow \frac{2R^2}{400} = 1 \Rightarrow R^2 = 200 \Rightarrow R = \sqrt{200} = 10\sqrt{2}$.

médiane :)

$$p(x) = c(a - |x|) \text{ si } x \in [-a, a]$$

$$\int_{\mathbb{R}} p(x)dx = \int_{-\infty}^{0} c(a+x)dx + \int_{0}^{+\infty} c(a-x)dx$$

$$\frac{2}{a^2}\int_0^a x^2(a-x)dx = \frac{2}{a^2}\int_0^a ax^2 - x^3$$

$$= \frac{2a}{a^2}\int_0^a x^2 - x^3 dx$$

$$= \frac{2}{a}\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^a$$

$$= \frac{2}{a}\left(\frac{1}{3}a^3 - \frac{1}{4}a^4\right)$$

$$=$$

$$(2a) \times a^3 = 2(a \times a^3)$$

associativité

de $\times$

$$\begin{cases} x^0 = 1 \\ x^{n+1} = x^n \times x \end{cases}$$

$$p(x) = a\int_1^2 \frac{1}{x^5}dx = a\left[\frac{x^{-4}}{-4}\right]_1^2 = a\left(\frac{2^{-4}}{-4} - \frac{1^{-4}}{-4}\right)$$

$$= a\left(\frac{1}{64} + \frac{16}{64}\right)$$

$$\frac{u'v - uv'}{v^2}$$

$$= \frac{15}{64}a = 1 \Rightarrow a = \frac{64}{15}$$