TD 9 : Proba.

Soit X une V.A continue.

Une fonction $f_X(x)$ est une densité de proba de X tel que :

$$\begin{cases} f_X(x) \geq 0 \quad \forall x \in \mathbb{R} \\ \int_{\mathbb{R}} f_X(x) = 1. \end{cases}$$

$$\mathbb{E}(X) = \int_{\mathbb{R}} x f_X(x)\,dx$$

$$Var(X) = \int_{\mathbb{R}} (x - \mathbb{E}(X))^2 f_X(x)\,dx.$$

Si $X \sim [a,b]$, la densité de proba de x est :

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0 & \text{sinon}. \end{cases}$$

$$\mathbb{P}(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$$

soit T le temps d'attente du passager, T suit une loi uniforme

$T \simeq U[0;15]$

$$\mathbb{P}(T \leq 5) = \frac{5-0}{15-0} = \frac{1}{3}$$

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 15 \\ \hline 225 \end{array} \quad 2$$

$$\mathbb{P}(T \geq 10) = 1 - \mathbb{P}(T \leq 10) = 1 - \frac{10}{15} = 1 - \frac{2}{3} = \frac{1}{3}.$$

$$\mathbb{E}(T) = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \cancel{\int_{-\infty}^{0} x f_X(x)\,dx} + \int_0^{15} x f_X(x)\,dx + \cancel{\int_{15}^{+\infty} x f_X(x)\,dx}$$

$$= \frac{1}{15}\int_0^{15} x\,dx = \frac{1}{15}\left[\frac{x^2}{2}\right]_0^{15} = \frac{225}{30} = 7.5 \text{ min.} \quad :)$$

$T \sim U[1;6]$

$$\mathbb{E}(T) = \frac{1+6}{2} = \frac{7}{2} \qquad \mathbb{P}(X \geq 5) = 1 - \mathbb{P}(X \leq 5) = 1 - \frac{6-5}{6-1} = \frac{1}{5}$$

$$E(f(X)) = \int_{\mathbb{R}} f(x) f_X(x)\,dx.$$

$X \sim U[1;3]$

$$\mathbb{E}(X) = \int_{\mathbb{R}} x f_X(x)\,dx = \frac{1}{2}\int_1^3 x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_1^3 = \frac{1}{2}\left(\frac{9}{2} - \frac{1}{2}\right) = 2.$$

$$\mathbb{E}(X^2) = \int_{\mathbb{R}} x^2 f_X(x)\,dx = \frac{1}{2}\int_1^3 x^2\,dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_1^3 = \frac{1}{2}\left(\frac{27-1}{3}\right) = \frac{26}{6} = \frac{13}{3}.$$

$X \sim \mathcal{E}(\lambda)$

$$\mathbb{E}(e^{X/2}) = \int_{\mathbb{R}} e^{x/2}\,\lambda e^{-\lambda x}\,dx$$

$$= \lambda\int_{\mathbb{R}} e^{x/2-\lambda x}\,dx = \lambda\int_{\mathbb{R}} e^{x(\frac{1}{2}-\lambda)}\,dx$$

$$= \frac{\lambda}{\frac{1}{2}-\lambda}\left[e^{(1/2-0)x}\right]_0^A \xrightarrow[+\infty]{} \frac{\lambda}{1/2-\lambda}$$

$u = x\left(\frac{1}{2}-\lambda\right)$ $\quad \boxed{0 \leq \lambda \leq \frac{1}{2}}$

$du = \frac{1}{2}-\lambda\,dx$

$$\int_{\mathbb{R}} \rho(x) = \int_{\mathbb{R}} cx\,\mathbb{1}_{[0,1]}(x) + (2-x)\,\mathbb{1}_{]1,2]}(x)\,dx$$

$$= \int_0^1 cx\,dx + \int_1^2 2-x\,dx$$

fais max :)

$$= c\left(\left[\frac{x^2}{2}\right]_0^1 - \int_{-2}^{0} x\,dx\right)$$

$$= c\left(\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^2}{2}\right]_{-2}^0\right)$$

$$= c\left(\frac{1}{2} - -\left(\frac{4}{2}\right)\right) = c\,\frac{5}{2}$$

$$c = \frac{2}{5}.$$